UTILITY OF FLIGHT DATA IN CALIBRATING ENGAGEMENT SIMULATIONS

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UTILITY OF FLIGHT DATA IN

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

This research presents and applies a proposed methodology for using test data in calibrating simulations. The method is applied to an existing simulation of an antiaircraft artillery fire control director with markedly successful results.

The first step in the proposed method is to develop hypotheses describing differences between the simulation and the actual system. In the application, three major hypotheses were developed. The one ultimately accepted was that the actual system included velocity and acceleration drag which was not modeled by the simulation.

The next step is to express each hypothesis in such a manner that they may be tested by consulting available flight test data. In the application, a regression model was constructed that would describe drag effects using azimuth and elevation tracking data. The independent variables were tracking velocity and acceleration of the simulation system. The horizontal (azimuth) and vertical (elevation) angular position errors between the simulation and actual systems served as dependent variables.

The third step is to test each hypothesis. Data from four test flights were used in estimating the regression parameters for the application problem. Data from another seven flights was used to test and demonstrate the prediction capability of the

technique.

Finally, the proposed method requires either a return to hypothesis development or closure in the form of substantial improvement in simulation accuracy. In the application problem, application of proposed corrections increased the percentage of simulated azimuth position errors that were less than 1.5 degrees from 24% to 100%. Likewise, the percentage of elevation position errors less than 1.5 degrees increased from 84% to 99%.

The research concludes that the proposed calibration technique did provide greatly improved simulation accuracy for the example of the study, suggesting that the technique may be useful for other calibration applications, especially in the area of military engagement simulations.

CHAPTER I

INTRODUCTION

Simulation programs that model engagement of aircraft by ground-based anticraft artillery often show credibility-limiting non-robustness when exercised over wide conditions. For example, three programs (P001, EVADE, and SIMFIND) can determine hit probabilities for a given problem such as one aircraft against one gun but they produce significantly differing results [2]. Such differences exist even though these simulations were constructed using common intelligence data and were validated using standard techniques [14]. As a result, military analysts cannot confidently use these simulations to plan force structures and deployment strategies.

Possible solutions to correct such differences are directly related to knowledge about the system being simulated. At one extreme, little may be known about a foreign (hostile) threat system that has never been seen in action. The modeler may only know the system's intended purpose during a military engagement. At the other extreme, essentially complete knowledge may be available because the system was designed by friendly forces. There are various levels of knowledge between these extremes. For example, scientific and technical knowledge of the hostile system increases with time (e.g., field tests on captured equipment, defector debriefings, actual engagement experience) and thus there are often additional data, not originally taken into account, constituting an improved technical data base for improving the simulation.

The simplest and weakest improvement procedure is pure calibration which consists of adjusting parameters empirically to cause the simulation output to more closely approximate observations. The most complicated and strongest improvement procedure is major remodeling -- restructuring the simulation in the light of new knowledge. For most simulations, one would expect the best approach to fall between these two extremes.

Two possible intermediate approaches are semi-empirical calibration and logic patching. The former consists of using intuition or facts concerning the differences between the simulation and the actual system to contrive a calibration technique that compensates for the differences. The latter approach, logic patching, includes minor computer program changes made without restructuring the program.

This research considers situations in which an already-developed and validated simulation yields inaccurate results, and there are available some data (field test data for example) that were not considered in original development of the simulation and shows promise of being useful in calibrating or altering the simulation model so that it will yield more accurate results.

Previous experience of the author with such situations indicates that it is usually not fruitful simply to choose one of the above approaches in advance and apply it to a given problem.

Neither is a scattershot application of all the approaches likely to be beneficial. The method proposed here is a formalization of what the author has found to be a workable procedure:

1. Hypotheses generation. Generate one or more hypotheses that attempt to explain the discrepancies between simulation and reality. The strongest (and therefore the most easily refuted) hypotheses are usually those of the form " X is present in the actual system but absent in the simulation, and proper incorporation of X into the simulation will significantly reduce error" where X is a quantifiable characteristic of the system.
2. Formulation. Express each hypothesis in a form so that test data can be used to support or refute it. This is a nontrivial task, since the analyst is not free to design an experiment. The test data already exist, and the challenge is to formulate hypotheses in such a way that the existing data support or refute them.
3. Testing and closure. Test the hypotheses. Upon success, stop. Upon failure or partial success, return to hypothesis generation.

This method of iterative hypothesis generation and testing is not original, but is simply a particularization of the modern scientific method as expounded by science historians such as Kneller [8]. The entire procedure from hypothesis generation to

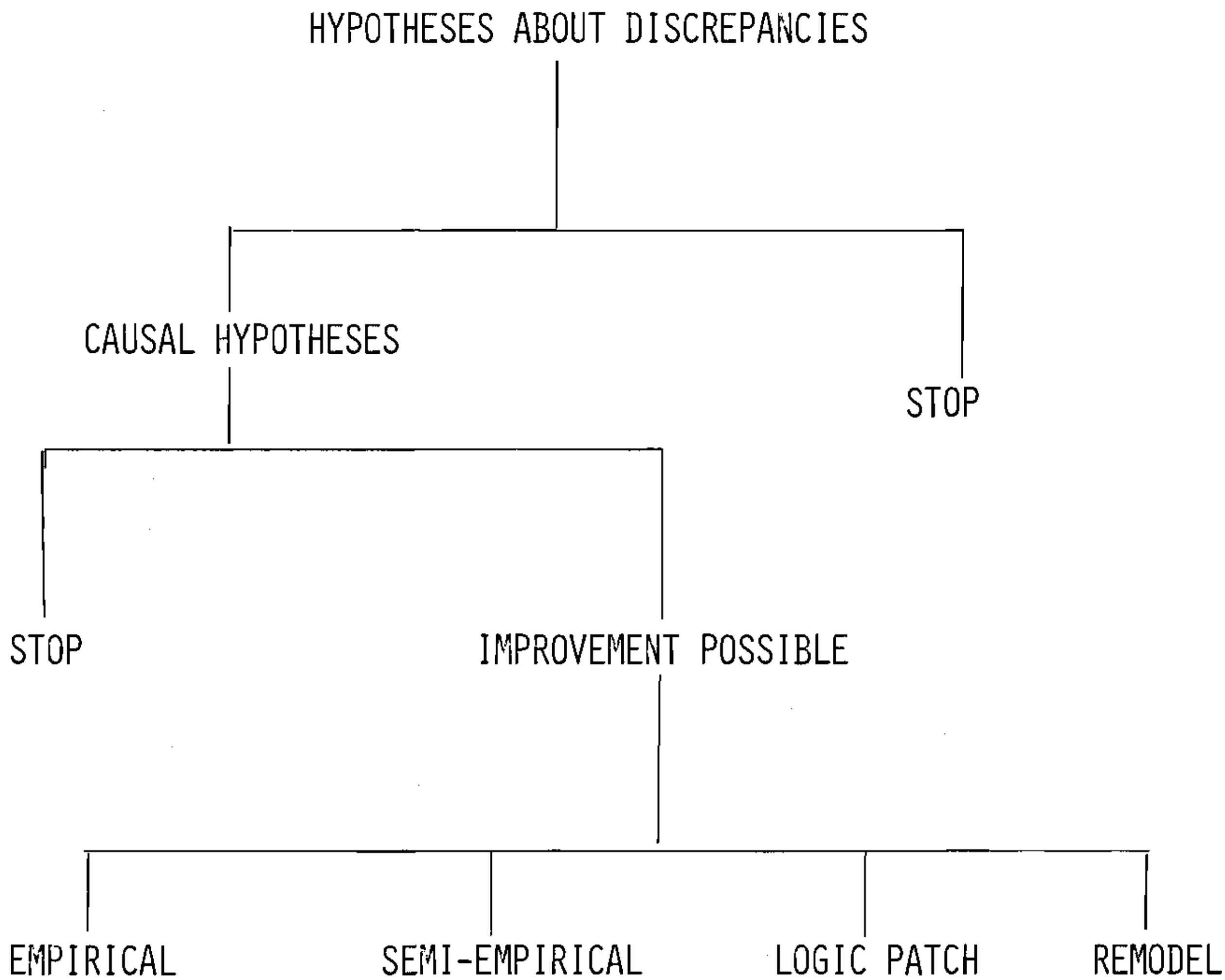

Figure 1. Summary of Hypotheses Generation to Improvement Approach

improvement approach is summarized in Figure 1.

The main task undertaken in the work reported herein was to use the above method to attempt to improve the accuracy of a simulation developed at Eglin Air Force Base to study performance of an antiaircraft weapon. The developers had realized that the simulation was giving inaccurate results, and it was suspected that the defect lay in the representation of the fire control computer.

Application of the method resulted in rejecting all hypotheses concerning misrepresentation of the fire control computer, and in accepting a later hypothesis concerning lags in the pointing system of which the computer is a part. The improvement corresponding to the selected hypothesis was a semi-empirical calibration in which turret movement was corrected for frictional and inertial drag, the correction amounts being calculated statistically from the flight data by regression techniques.

1.1 Objective and Procedure

The primary objective of this research is to demonstrate the iterative method of hypothesis generation and testing to obtain a more accurate simulation by use of field test data. A secondary objective is to demonstrate that corrective parameters may be obtained by multiple regression techniques under a causal hypothesis.

This thesis is organized as follows. The system being simulated will be described. Hypotheses concerning differences between the system and its simulation will be discussed. Regression equations will be formulated, and their robustness and variance reduction will be studied to test causal hypotheses and to obtain correction parameters under the accepted hypothesis. Finally, the thesis will examine the trade-offs between the benefits of improved simulation prediction accuracy and the cost of using additional calibration data.

1.2 Review of Literature

Validity of a computer simulation model as described by Shannon [14] is the "level of confidence that inferences drawn from the performance of the model are correct and applicable to the real-world system." Shannon further indicates that there are degrees of validity of a model and that validity is not a yes-no variable. Determining the degree of validity is one of several steps performed in completing a computer simulation [3].

Naylor [3, 10, 11] views validation as consisting of three types of verification. The first is rationalism which assumes that verification is a "problem of searching for a set of basic assumptions underlying the behavior of the system of interest." Empiricism, the second type which is opposite to rationalism, holds that all assumptions made concerning the system must be supported directly by observations. The third type, positive economics, rests on the ability of the model to predict the behavior of the dependent variables which are treated by the model. Naylor also discusses multi-stage verification, which is a composite of the preceding three types. Multi-stage verification first "formulates a set of hypotheses describing the behavior of the system of interest." Next, the hypotheses are verified using available statistical techniques. Finally, the prediction capability of this simulation is tested. If all tests support the original hypothesis, the simulation is considered validated. Otherwise, some appropriate model changes are made and the process is repeated.

There is no literature on calibration as such for model validation, because it is simply a low-effort form of modification to obtain increased empirical validation. In this research, the term calibration is defined as the act of changing simulation parameters so that a measure of error between the simulation and real system is minimized. Distinguishing calibration from validation is necessary since the technique described by this research may be applied after a modeler has finished validating the simulation.

In practice, there is a spectrum of methods for demonstrating the degree of validity (or conformity) between the simulation and the real system. At the purely intuitive or subjective end are comparisons like the "Turing Test" [14,16] which involve letting experts attempt to distinguish between input-output data of the simulation and the real system. At the other end (objective) are strictly mathematical tests like analysis of variance and non-parametric tests (e.g., Kolmogorov-Smirnov test, Chi-Squared test, etc.). Naylor [9] has also discussed several methods which fall between these extremes. Purely graphical analyses [4] lies close to the intuitive methods in that the analyst observes graphs of time series or other performance parameters and makes subjective statements (e.g., trend statements) concerning the model's validity. Theil's Inequality Coefficient [9, 15] is another intermediate method that provides a measure of the degree of prediction accuracy to observed historical data.

The selection of a particular test for validation should be based on the accuracy of performance required of the simulation (i.e., the accuracy required for its intended use) and on the cost of validation. There will normally be a trade-off between increasing the validity and the cost of increased validation [1]. In some cases statistical tests may not be required if it is readily obvious that the model does or does not fit the real system. Statistical tests are required when the agreement or fit is not clear or if the modeler wishes to substantiate his intuitive findings in a quantitative manner. In the following research effort, the graphical method has been heavily used because this clearly demonstrated velocity and acceleration discrepancies between the simulated and actual systems.

CHAPTER II

METHODOLOGY

2.1 Introduction

This chapter describes the method used to calibrate program P2418, an antiaircraft artillery fire control director computer simulation[1]. The first step produces hypotheses that explain possible differences between the actual system and P2418. Two of these hypotheses were inadequate and were quickly rejected. The third hypothesis which has both intuitive and analytic appeal was used in developing a multiple regression model. Finally, a hypothesis testing procedure is presented. The procedure consisted of selecting flight test data passes that were used for calibrating P2418 and demonstrating their robustness. A detailed description of the flight test is presented in Appendix A.

2.2 Hypothesis Generation

The hypotheses were developed by first considering the block diagrams of the two systems, Figure 2. For the purposes of this study, the actual system consists of a target-tracking radar, the fire control director computer, and gun positioning servos.

In operation, the fire control director's analog computer uses the target aircraft position data and projectile ballistic

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1 This program was developed by the Directorate of Computer Science, Eglin AFB, Florida.

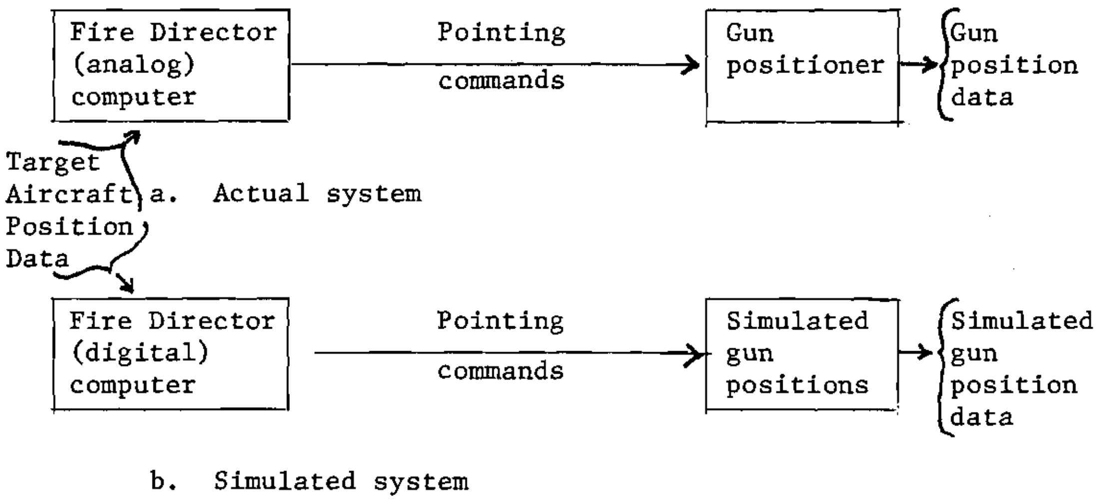
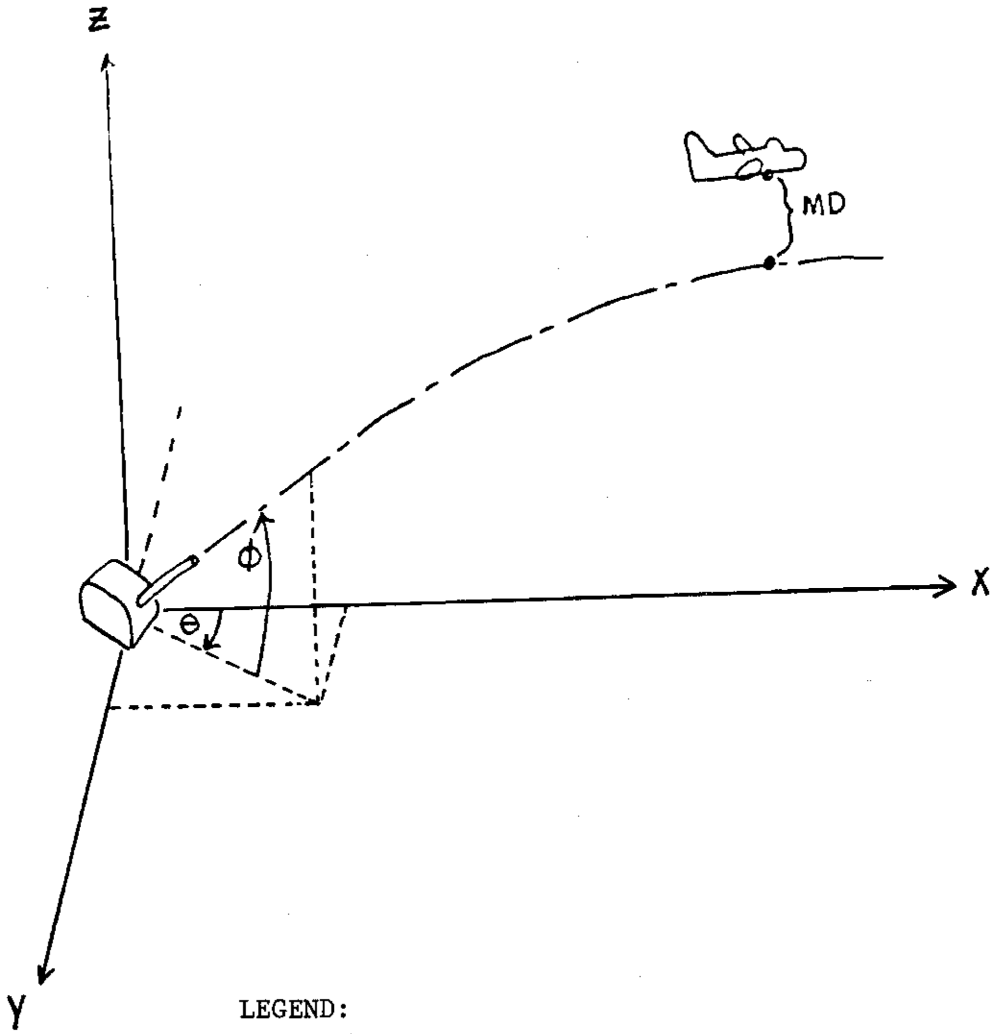


Figure 2. Block Diagram of Actual and Simulated Systems

information to compute required gun positioning commands that will cause the projectile to intercept the target aircraft (see Figure 3). The computer must also make adjustments for wind, target velocity, gun servo lead/lag, gun inertia, turret inertia, etc.. The simulated system in the lower half of the diagram consists of a digital simulation of the fire control director's analog computer and the simulated gun servos. Note that the simulation does not simulate the tracking radar. Aircraft position data can be provided to the simulation by the actual radar or another target tracking radar. This system design feature allows isolation of the fire control system as a subsystem.

The two systems' gun position data were compared using the actual system's target aircraft position data. Since the projectiles cannot be redirected once they leave the gun, the gun position



LEGEND:

$$MD = MIN \{\sqrt{(X_1 - X_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}\}$$

X_1, y_1, z_1 = Aircraft Position

X_2, y_2, z_2 = Projectile Position

ϕ = Elevation Angle

Θ = Azimuth Angle

Figure 3. Engagement Geometry

at time of projectile fire suffices in measuring the performance difference between the systems. Specifically, these data constitute the gun azimuth and elevation time series and consist of gun azimuth and elevation positions versus time. A special computer program was developed to compute the differences between the model and measured time series (errors) and on scanning several data passes it was observed that the largest magnitudes of error occurred when the angular velocities were greatest. The azimuth errors of one such data pass are presented in Figure 4.

Similar errors were obtained for elevation angles, but they were smaller in magnitude. It was observed that the simulation was "overpredicting" or producing gun positions that led the actual gun positions. Furthermore, these errors increased, at given azimuth velocity, as the azimuth acceleration increased. This fact is evident from the azimuth velocity and acceleration differences (errors), Figures 5 and 6.

These comparisons suggested three possible hypotheses. The first hypothesis was generated using these comparisons and a model description of a similar simulation presented in a paper [2] which contained a more detailed simulation of the gun positioning servomechanisms than P2418. The hypothesis was that including this additional detail in P2418 would correct the "overpredicting" experienced in the current model. Eglin Air Force Base personnel incorporated these details into the model and re-ran the data. Since negligible difference in "overpredicting" was evident, this

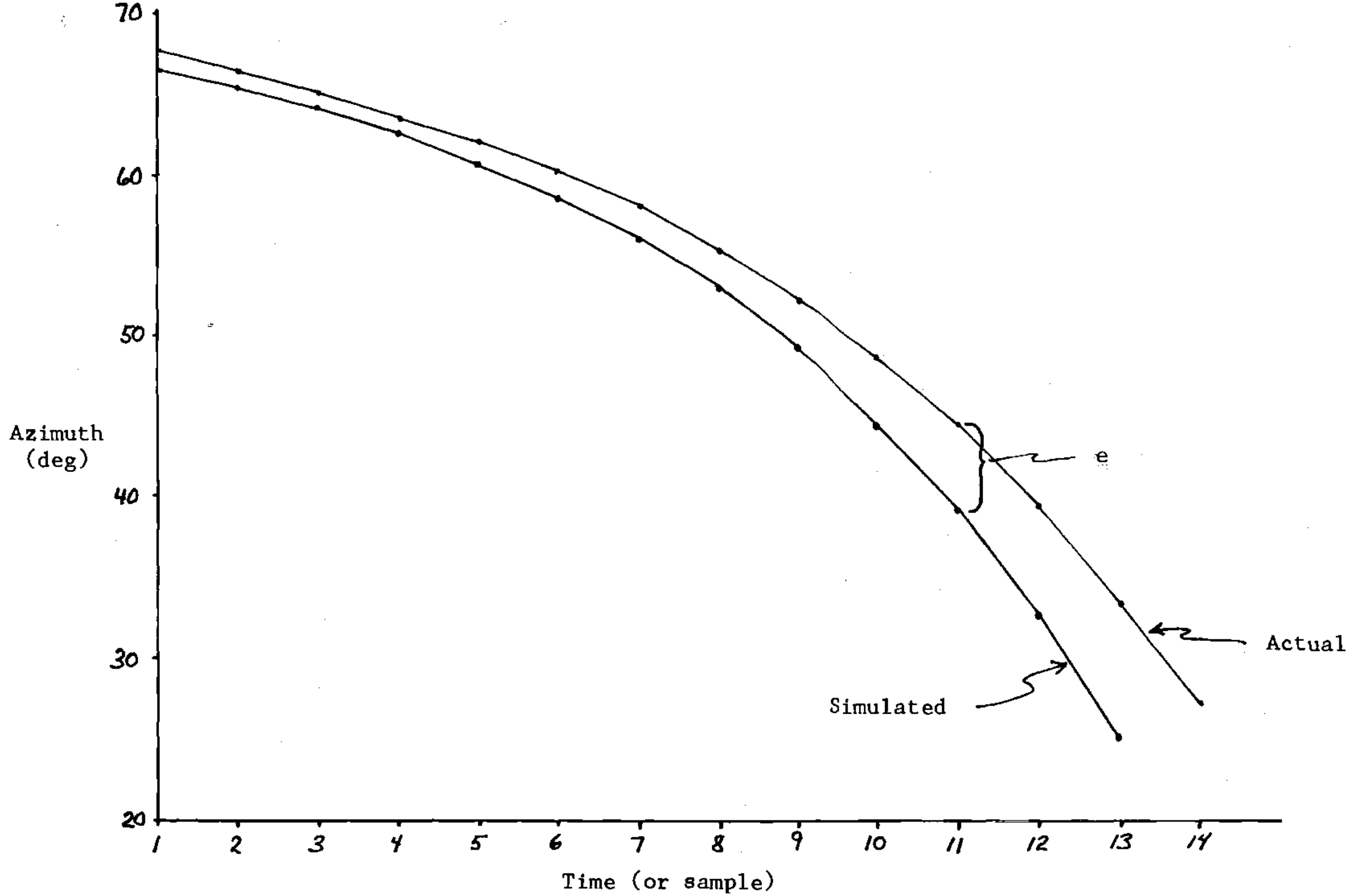


Figure 4. Azimuth position for actual and simulated systems.

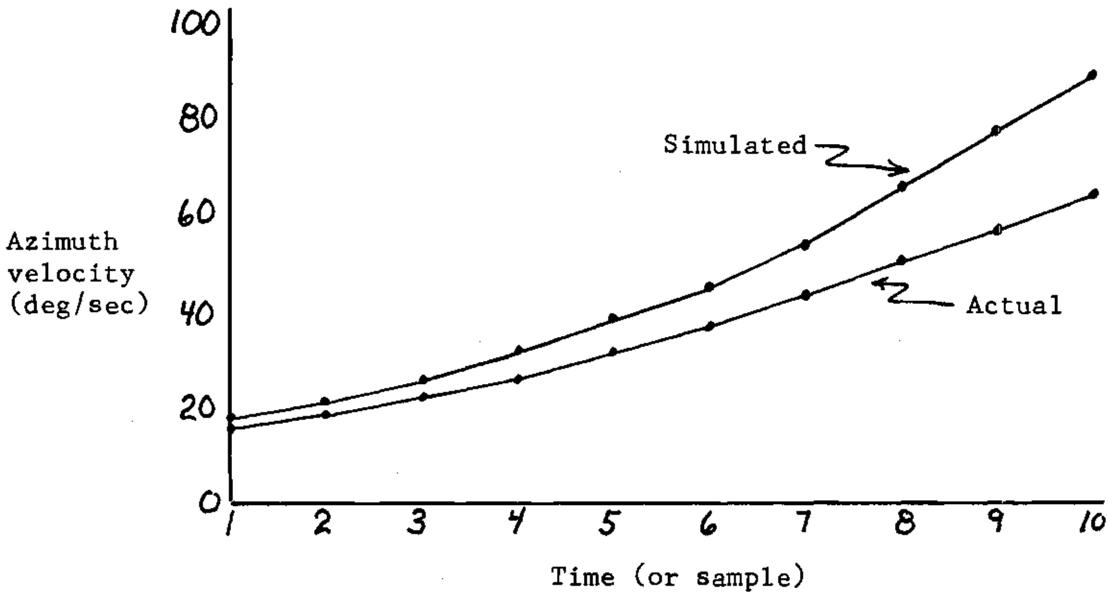


Figure 5. Azimuth velocity for actual and simulated systems.

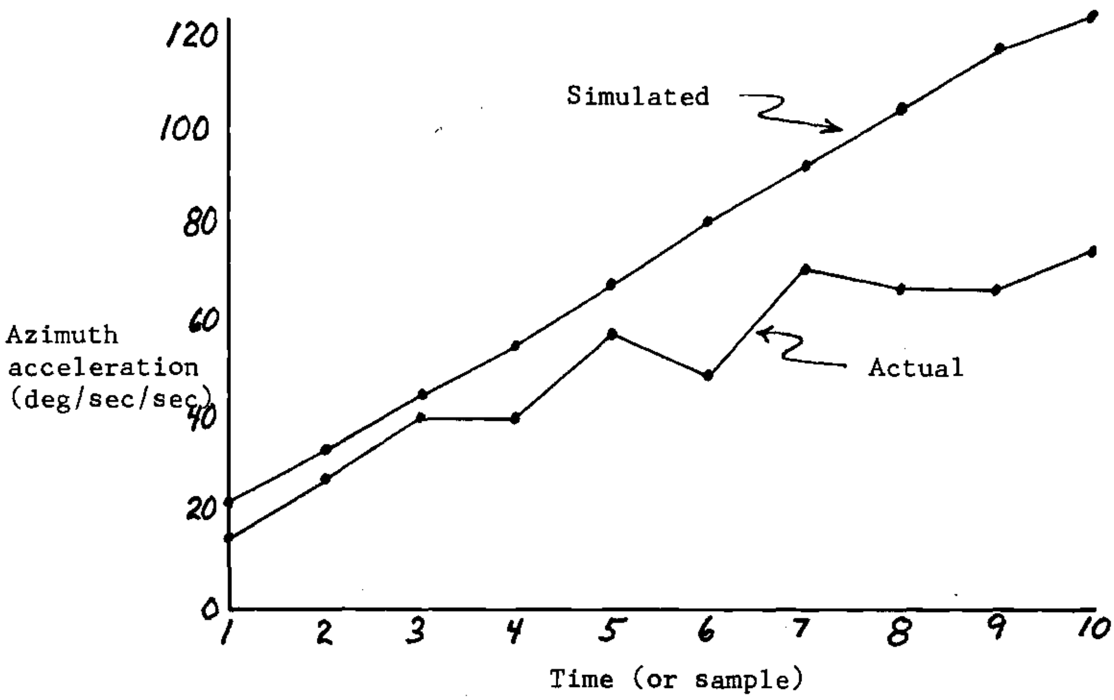


Figure 6. Azimuth acceleration for actual and simulated systems.

hypothesis was rejected.

The second hypothesis related to the possible differences in the digital simulation of the analog fire control director computer's functional operation. The portion of the simulation in question was the gun positioning commands obtained by solving a set of target prediction equations using z-transformation technology. In the simulation these equations were solved at each sampling instant by iterative computation of the target prediction until a certain convergence was obtained. In the analog computer these equations were solved continuously using the computer's electrical, mechanical and electromechanical components. The hypothesis was that the analog computer was obtaining only partial convergence of the solution during the time interval that the digital simulation was obtaining complete convergence. The attempt to incorporate this lag into P2418 consisted of making only one iteration of the prediction equations solution for each sampling instant. The results using the same data passes indicated some improvement but not enough to warrant pursuing this course of action.

The third hypothesis suggested by the comparisons was that the actual system exhibits more drag[1]-- both frictional or "velocity" drag and inertial or "acceleration" drag -- than the

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[1] A "drag" is a force opposing the force exerted by a servo-mechanism. A "lag" is a position error, one cause of which can be that the net force (servo force minus drag) is not large enough in comparison to the rotational inertia to move the gun servo system as quickly as required.

simulated system. This hypothesis had immediate appeal because (1) modern friendly weapons-positioning servo-mechanisms are designed to render frictional and inertial lags negligible, so that lags might have been neglected in the P2418 simulation model; (2) the actual weapons-positioning system seemed undersized and underdesigned, so that appreciable lags of these types might possibly be expected; and (3) the actual director analog computer, having motors subject to slippage and gears subject to backlash, friction, etc. could be expected to have lags in its output commands.

It was quickly ascertained that the simulation model indeed contained no simulated lag due to these drags; thus, point (1) above was strengthened. Point (2) was supported by the fact that similar friendly systems have larger servo-mechanisms and are much more responsive to positioning commands.

The simplest approach to calibration consistent with the above hypothesis and its motivation would be to retard the simulated gun position by an amount proportional to the velocity and by an additional amount proportional to the acceleration, according to well-known principles of mechanics [13]. This would assume constant moment of inertia of the rotating mass and constant force from the servo-mechanism's drive motors and also similar drag behavior of the components of the analog computer, with additivity of the lags from computer drag and physical drag.

If this calibration did not show acceptable results, it was planned to revise the hypothesis by taking into account such things as changes in inertia effects of the gun/turret system, and to investigate more closely the separate lags in the computer gun commands and the guns resulting positional data.

2.3 Regression Model

A multiple linear regression model was selected to represent the proportional relationship between the actual and simulated gun positional data [5]. The differences between the two systems' positional data, or the amount of simulation position retarding necessary, was the dependent variable, e. This actual difference can be seen in Figure 4 as the position error. The simulated gun velocity and acceleration were used as the independent variables for the regression and were time correlated with the position error. Note that the actual gun velocity and acceleration data were not selected as the independent variables since these would not be available when using the simulation with the regression model for predicting actual gun positions. The proposed model was:

$$e = b_0 + b_1X_1 + b_2X_2 + \varepsilon$$

where e: Dependent variable

b_0, b_1, b_2: Constant coefficients to be determined

X_1: Velocity

X_2: Acceleration

ε: Noise

Since the system being simulated was required to track in either direction, it is reasonable to assume that the overall bias contribution was zero. In addition, possible instrumentation bias was removed during data reduction by calibration (Appendix A). Consequently, the general regression equation was modified by removing the b_0 constant which implies that the regression line passes through the origin [12].

The discussion to this point has only addressed azimuth angle errors, but in the actual and simulated systems there were also similar elevation angle errors that were modeled. These two types of errors can be modeled separately because they represent the output of separate orthogonal systems. (The output is orthogonal because the fire control director solves the position commands after target position has been projected onto the horizontal plane for azimuth and the vertical plane for elevation.) Once the radar input has been received by the fire control director computer, Figure 2 separate fire control computing mechanisms compute new commands that are sent to separate azimuth and elevation drive servo motors.

Consider the simulated azimuth position command time history given in equal time intervals (e.g., for a sampling interval of I = 0.10 second). These angle positions can be represented by the time series $\ldots x_{i-1}, x_i, x_{i+1}, \ldots$ where x_i is a particular position at time t_i. The velocity and acceleration can be obtained by taking the first and second difference of this series, respectively, and dividing by the sampling interval (I). For example, the velocity at time t_i is $v_i = (x_i - x_{i-1})/I$. The acceleration at time t_i is $a_i = (v_i - v_{i-1})/I$. Let ... x'_{i-1}, x'_i, x'_{i+1}, ... represent the azimuth position commands of the actual system. Since the position commands were recorded at the same time t_i, the error at time t_i is $e_i = x_i - x'_i$. The corresponding velocity and acceleration data used to develop the regression equation would be v_i and a_i respectively. Now after selecting enough data to produce the final regression equation, the adjusted simulation position command, $\hat{y}_i$, can be obtained by subtracting the estimated error, $\hat{e}_i$, from the simulation position commands, y_i, using the simulation estimates of velocity and acceleration.

2.4 Selecting Data Passes for Hypothesis Testing

Data passes -- test aircraft flights over the actual artillery system -- provided a means of testing the hypothesis by first provisionally calibrating the simulation and, then determining the prediction capability of the calibrated simulation program. This was accomplished by partioning available historical data into two sets. The first set was used to calibrate the simulation; the second set was used to assess the simulation's prediction capability.

In order to demonstrate the power of using this technique for calibration, as few passes as possible should be used for calibration. This set of passes could be determined by selecting passes until the set includes data from the entire range of velocity/acceleration combinations experienced on all possible passes. This selection criterion would minimize the amounts of extrapolation necessary when computing the prediction errors. Some redundancy will be necessary to account for pass-to-pass variations, but it will be assumed that the time series data for passes is ergodic and stationary. Passes should be selected with both clockwise and counterclockwise tracking to counteract any effects of bias due to lack of left-right symmetry in gear lag, boresight errors, etc.

Figure 7 shows the comparison of azimuth position, velocity and acceleration between the actual and simulated systems for a typical pass of data. Note that only the last 25 samples or 2.7 seconds have been plotted since the preceding points had small errors. This figure also illustrates a limiting restriction of the actual system that starts around sample 15. The azimuth velocity clearly demonstrates this. This limiting factor can be easily modeled by setting the simulation rates to the maximum of the actual system, about 80 degrees per second, when the predicted rates exceeded this maximum. This study assumes this trivial correction and addresses only the inbound portions of passes until the velocity rates were exceeded. The elevation data were also considered only up to sample 15 (although the elevation data did not exhibit such limiting rates).

Further evidence of the spread in sample values is presented in Figure 8 where the position error is shown as a function of

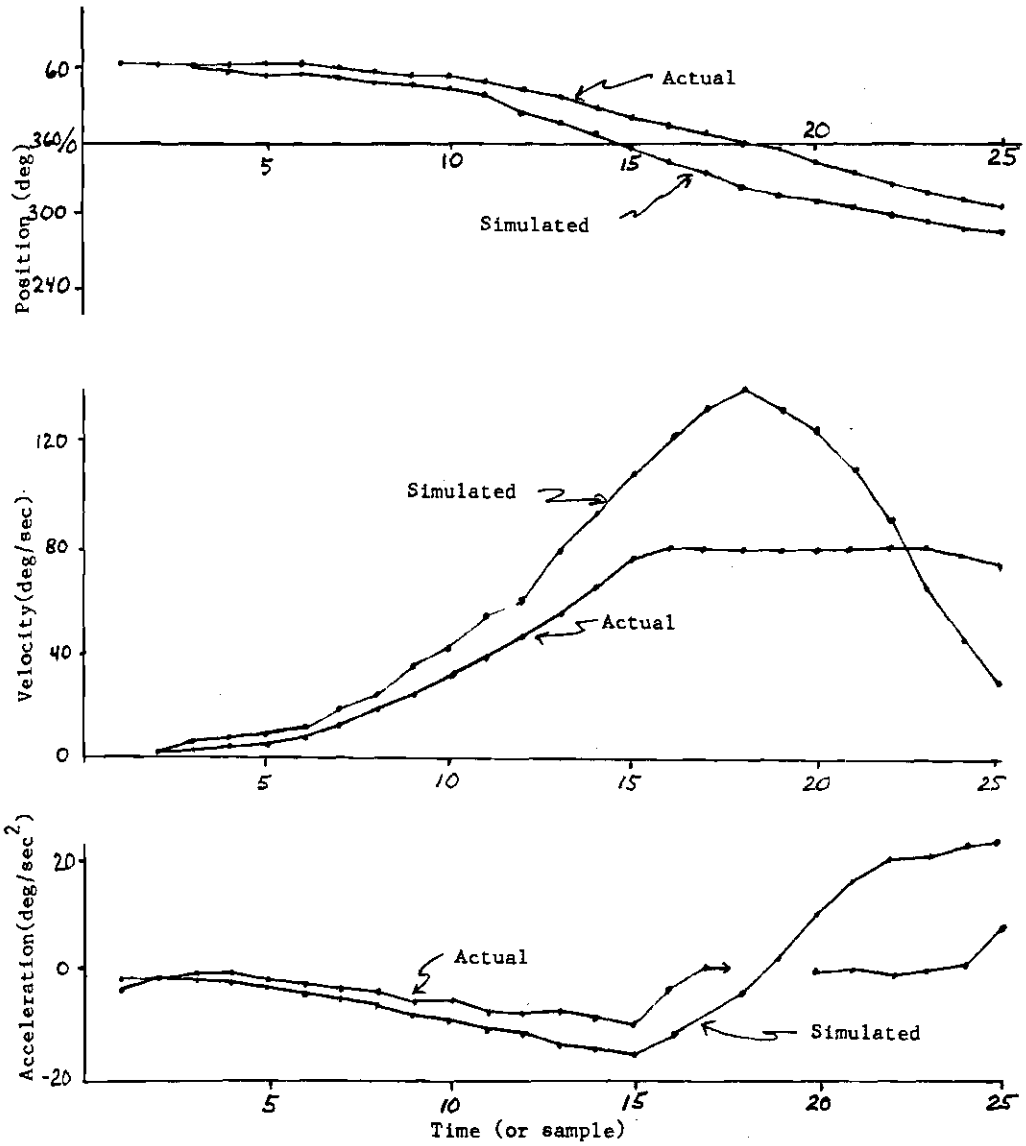

Figure 7. Typical Pass Characteristics

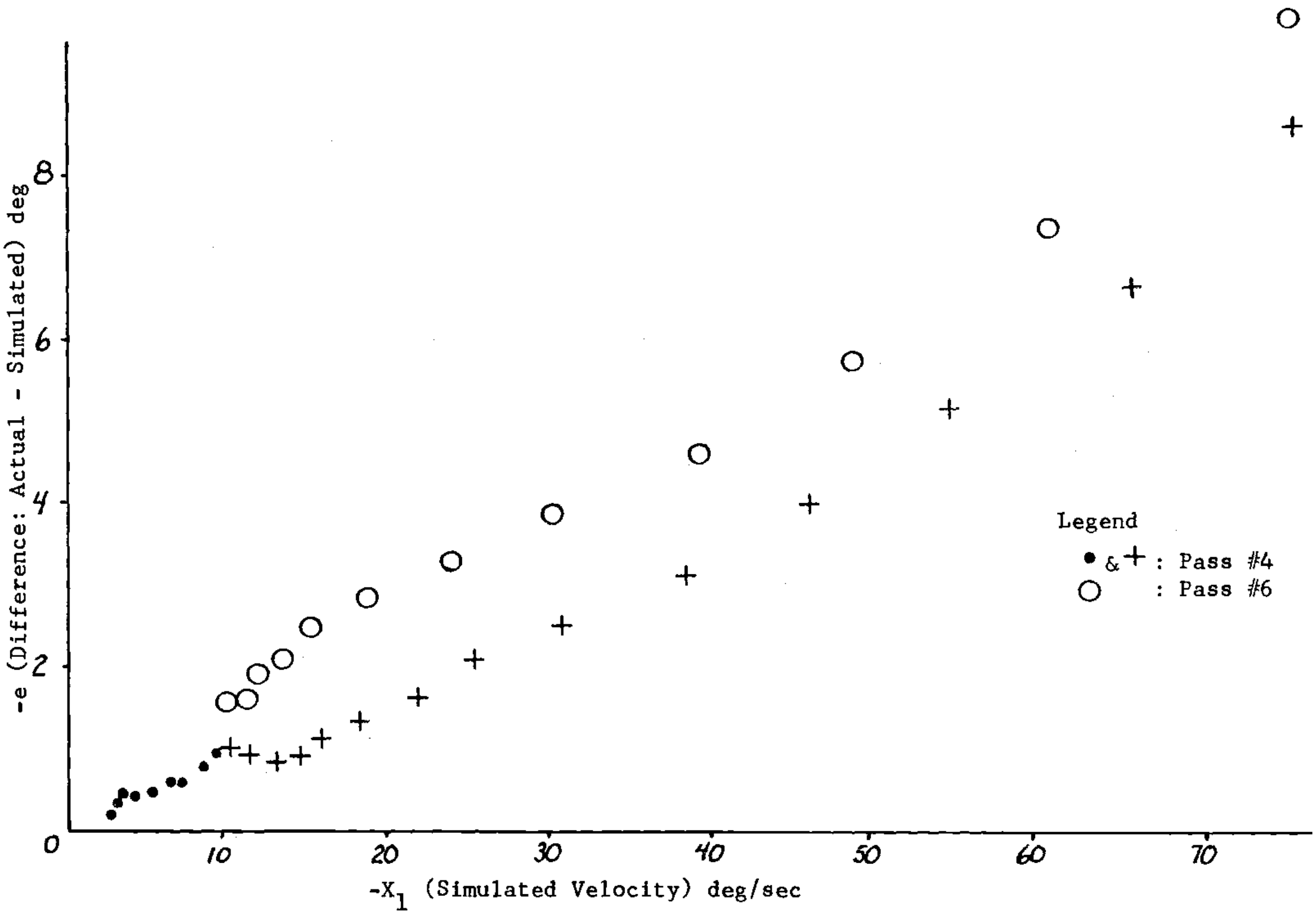

Figure 8. Velocity Effects on Position Error (Azimuth)

azimuth velocity for a generic data pass. Note that only fifteen of the samples have velocities larger than 10 degrees per second. Also note that the position error remains less than one degree until the velocity exceeds 15 degrees per second.

In view of this discussion, only the relevant portions of data passes were used in the parameter estimation equation. Specifically, data were only included from the last parts of passes when the simulated velocity exceeded 10 degrees per second and the actual system velocity did not exceed 80 degrees per second. Four passes, 4 through 7, were selected as candidates for constructing regression equations. Passes 4 and 6 were expected to produce better results than 5 and 7 since the former passes contained higher velocity data (Table 1). The remaining passes would be used for demonstrating the prediction capability.

The sum of squared error and the maximum error were used as measures of goodness of fit. From a previous study [17], only prediction errors of less than 1.5 degrees could be tolerated based on the projectile miss distance error that would be attributed to instrumentation. A 1.5 degree azimuth error with the target aircraft at 2,000 feet range corresponds to approximately 50 feet of error in miss distance.

Table 1. Maximum Angle Velocities Attained

	Maximum Velocity (degrees per second)	
Pass	Azimuth	Elevation
2	4	20
3	-23	-8
4	-90	15
5	-19	-6
6	-95	16
7	-19	-6
8	-95	16
9	-19	-6
10	-95	18
11	-20	-6
12	-30	16
13	-20	-7
14	-21	-7
15	-20	-7
16	-21	-7
17	0	20
18	-18	-5

CHAPTER III

RESULTS AND DISCUSSION

3.1 Introduction

The three areas covered in this chapter include the procedure used to develop the final regression parameter estimates, the capability of the model to predict the simulation errors for passes not used in the regression model construction, and, finally, the utility associated with this regression technique from both technical and economic viewpoints. The discussion will not be limited to this specific work, but will be slightly more general in order to provide insight whereby the regression technique may be useful for similar applications. The last section on utility especially fits into the more general category because it presents trade-offs that should be considered when using this technique.

Let e_{az} and e_{el} denote position errors resulting from the simulation program, P2418, when compared to the actual system. Let $\hat{e}_{az}$ and $\hat{e}_{el}$ denote estimated simulation position errors calculated by regression. The differences $e_{az} - \hat{e}_{az}$ and $e_{el} - \hat{e}_{el}$ represent amounts by which the regression fails to predict errors e_{az} and e_{el}.

3.2 Parameter Development

The first step in selecting the four passes for use in computing regression parameter estimates was to classify passes according to aircraft heading (East or West), tracking direction (clockwise or counterclockwise), tracking velocities (high or low), and magnitude of simulation position errors, and to examine for error trends.

The first classification of aircraft heading should not have any effect on the simulation or the actual system. Table 2 indicates that the higher azimuth tracking velocities were attained on the westbound headings, but only to the extent that is fully attributable to the fact that the flight profiles westbound passed closer to the weapon (Appendix A).

The next classification, tracking direction, could be used for detecting possible biases relative to pass direction. In particular, this bias would be observed by noting larger simulation mean errors in one direction. (Recall that left-right symmetry was one of the assumptions of the regression model which required the intercept to pass through the origin). Unfortunately, all tracking was counterclockwise which made it impossible to look for this source of possible biasing. A hypothesis that some of the error was due to an out-of-level condition could not be tested.

The tracking velocity provided qualitative information on how much each pass would contribute to the regression parameter estimates. Since the model uses tracking velocity and acceleration as independent variables, only the passes with sufficiently high velocity and acceleration data would provide calibration in data. For example, pass 5 would not provide any calibration data above

19 degrees per second. Finally, the simulation position errors were tabulated to insure that high tracking velocity passes normally had large position errors. If the actual system had malfunctioned on low velocity passes, the large position errors would indicate that something was not consistent and the pass would be inspected further.

Using the above classifications and the censoring procedure mentioned in the previous chapter, several of the non-calibration passes were eliminated before doing any calibration or prediction. Pass 2 was eliminated because the azimuth tracking velocity never exceeded more than 10 degrees per second. Passes 10 and 17 were eliminated because the actual system lost track for some unknown reason just as the azimuth tracking velocity started increasing. A data reduction error led to duplicating pass 13 for passes, 14, 15, and 16 which were accordingly rejected. Pass 1 was eliminated because a tracking bias was not accounted for during data reduction.

The next step, after elimination of invalid passes was to select one or more of passes 4 through 7 for calibration (or regression parameter estimation). Obviously, at least one high and one low tracking velocity pass should be chosen to get a representative sample of the velocity range. As a consequence, passes 4 and 5 were selected with passes 6 and 7 set aside for possible inclusion. This set of four could be further expanded if necessary, but the objective was to keep the set as small as possible to support the idea that the regression technique should be powerful. It should be noted that this set of four passes comprised 40% of the available passes, leaving only passes 8 and 12 that had high azimuth tracking velocities.

The regression parameter estimates for the fifteen samples of pass 4 are presented in Table 3. The extremely large coefficient of multiple determination R^2, values and the time history plot of the data, Figure 8, suggest that the data are highly autocorrelated. Consequently, the estimated autocorrelation coefficient of order 1 was computed using [12]:

$$r = \frac{\sum_{t=2}^{n} e_{t-1}\, e_t}{\sum_{t=2}^{n} e_{t-1}^2}$$

where e_t is the residual error after the regression is applied to the original data

The resulting r was 0.65. According to Neter and Wasserman [12] with autocorrelated data the regression parameter estimates will not have the minimum variance property and may be inefficient. In addition, the student's t and F statistics are no longer valid and, thus, may not indicate exact contributions, or non-contributions, of independent variables. Also, the mean squared error, MSE, may be underestimated. As a result of this, the analysis of variance (ANOVA) tables were not strictly used as a guide in eliminating variables from the regression. In particular, the F statistic indicated that the acceleration variable provided little information, but the variable was left in all models for intuitive reasons.

The variance inflation factor was 76, which indicated multicollinearity between the independent variables. Figure 9, which is a plot of one independent variable against the other, definitely suggests multicollinearity. One effect of multicollinearity could be poor prediction capability for values of independent variables that do not fall close to those in Figure 9. As a consequence of the particular flight profile flown for this test, all variables fell close to these, even though in actual engagements other profiles such as close-in orbits would produce high azimuth velocity and small azimuth accelerations which would fall far from the data in Figure 9. Fortunately, the parameter values for the acceleration, b_2, were small for both azimuth and elevation so this should not present a great problem.

As mentioned above, the initial strategy called for estimating the errors of pass 6 using pass 4 parameters. Since these two passes were similar (Table 2), it was hoped that a good prediction would result. If the estimates were accurate, then these paramters would be used to estimate errors of other passes until all were estimated accurately or the pass 4 parameters failed. This procedure failed immediately with pass 6

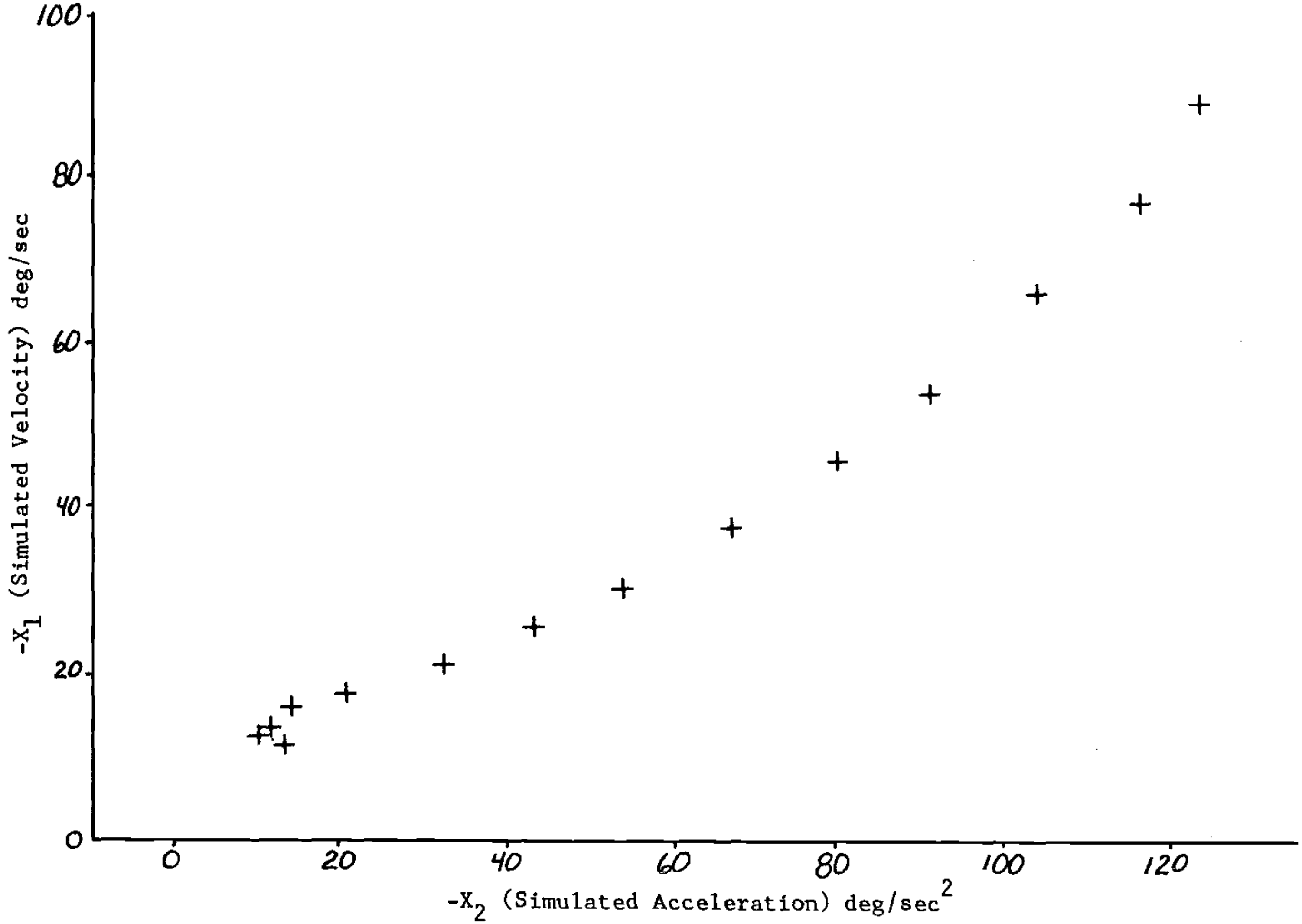

Figure 9. Independent Variables of Pass 4

Table 2. Flight Test Data Pass Summary

Pass	Use Calibration (c) Prediction (p)	Maximum Azimuth Velocity (deg/sec)	Maximum Errors (deg) Azimuth	Elevation	Samples Used
1[e]	a	----	---	---	---
2	b	---	---	---	---
3[e]	p	-23.4	0.8	-1.2	61
4	c	-89.2	-11.2	1.8	15
5[e]	c	-19.2	- 2.4	-1.5	56
6	c	-115.9	-18.1	2.4	14
7[e]	c	-19.4	-2.4	-1.6	58
8	p	-105.5	-14.9	1.9	13
9[e]	p	-19.1	-2.6	-1.6	52
10	c	---	---	---	---
11[e]	p	-20.5	-2.9	-1.5	57
12	p	-40.8	-4.3	2.4	6
13[e]	p	-21.2	-2.7	-1.7	56
14	d	---	---	---	---
15	d	---	---	---	---
16	d	---	---	---	---
17	c	---	---	---	---
18[e]	p	-18.8	-2.5	-1.2	57

a: Pass Used by Computer Sciences Laboratory for Mission Calibration
b: Azimuth Velocity Never More Than 10 deg/sec.
c: Actual System Not Tracking Well
d: Same Data as Pass 13, Mistake in Data Reduction
e: Eastbound Heading

predictions resulting in a MSE of 8.72. Passes 4 and 6 were studied further. Comparing azimuth velocities as shown in Figure 8 indicated enough difference between these passes to cause the poor predictions. This suggested that more information was needed to estimate the regression parameters. Consequently, it was decided to group all calibration pass data (passes 4 through 7) together and attempt to predict other pass errors.

3.3 Prediction Capability

Grouping data from passes 4 through 7 resulted in the following regression equations:

$$\hat{e}_{az} = 0.121\ X_1 + 0.003X_2 \qquad \text{(azimuth)}$$

$$\hat{e}_{el} = 0.139\ X_1 + 0.025X_2 \qquad \text{(elevation)}$$

The associated R^2 values were 0.96 and 0.55 for azimuth and elevation, respectively. There was some concern for the low elevation value and it deserves further comment.

Passes with elevation error varying by less than a degree resulted in low R^2 values (see Table 3, passes 5 and 7). These small variations, along with normally small elevation velocity data, did not provide enough data to accurately fit a regression equation to the relationship between the actual and simulated systems.

Another observation was made which did not affect the single pass regression but would affect multiple pass regressions.

Table 3. Regression Parameter Estimation

Data Pass(es)	Azimuth b_1	Azimuth b_2	Azimuth MSE[a]	Azimuth R^2	Elevation b_1	Elevation b_2	Elevation MSE	Elevation R^2
4	0.174	-0.045	0.49	0.98	0.125	0.005	2.01	0.98
5	0.123	-0.066	0.08	0.98	0.113	0.219	0.64	0.48
6	0.245	-0.054	0.28	0.99	0.003	0.002	0.03	0.98
7	0.120	-0.078	0.07	0.98	0.148	0.144	0.77	0.44
4,6	0.090	0.021	1.29	0.99	0.130	0.003	0.02	0.98
5,7	0.121	-0.072	0.07	0.98	0.135	0.171	0.070	0.45
4,5,6,7	0.121	0.003	0.38	0.96	0.139	0.025	0.73	0.55

a: Residual Mean Squared Error

Table 4. Prediction Comparisons (Simulation/Regression)

Pass	Samples Used	Sum of Squared Errors (deg) Azimuth	Sum of Squared Errors (deg) Elevation	Mean Squared Errors (deg) Azimuth	Mean Squared Errors (deg) Elevation
3	61	226.22/19.55	91.65/60.42	3.77/0.33	1.53/1.01
8	13	499.81/8.15	34.91/2.19	41.65/0.68	2.91/0.18
9	52	210.76/12.75	60.17/35.28	4.13/0.25	1.18/0.69
11	57	238.91/13.57	70.09/43.44	4.27/0.24	1.25/0.78
12	6	47.75/1.76	27.87/0.34	9.55/0.35	5.57/0.07
13	56	217.23/9.85	69.90/45.16	3.95/0.18	1.27/0.82
18	57	200.66/12.25	32.10/21.47	3.58/0.22	0.57/0.38
TOTAL	302	1641.34/77.88	386.69/208.30	5.45/0.26	1.28/0.69

Elevation position errors were always positive for eastbound headings and were always negative for westbound headings (due to instrumentation problems, according to personnel at Eglin Air Force Base), which could cause poor elevation predictions when using grouped pass data. Some adjustments might be made to increase the consistency of the data, but none were made. (This decision was supported by the fact that the regression technique always produced better results than the simulation.)

Position error predictions were computed for the seven passes indicated in Table 4. The comparisons in this table show that in all cases the mean squared errors were greatly reduced when using the regression technique. The elevation showed less improvement than the azimuth. The regression technique cumulative improvment was 95% for azimuth error and 46% for elevation error. Additional analysis of these data, Figures 10 and 11, indicate that all of the azimuth errors when using the regression technique were less than 1.5 degrees, compared to 24% using the simulation. Ninety-nine percent of elevation errors when using the regression technique were less than 1.5 degrees, compared to 84% using the simulation.

3.4 Utility

This section discusses the utility of using the regression technique from the technical and economic viewpoints based on the findings of this study. Factors such as regression accuracy, minimal data set requirements, instrumentation accuracy, and

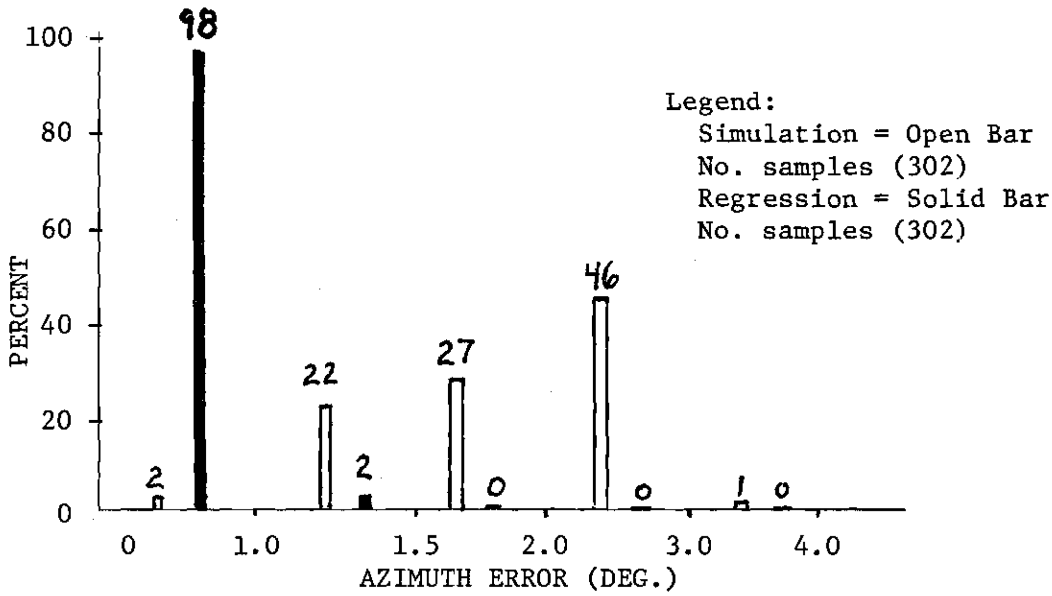

Figure 10. Absolute Azimuth Error Distribution (Prediction Passes)

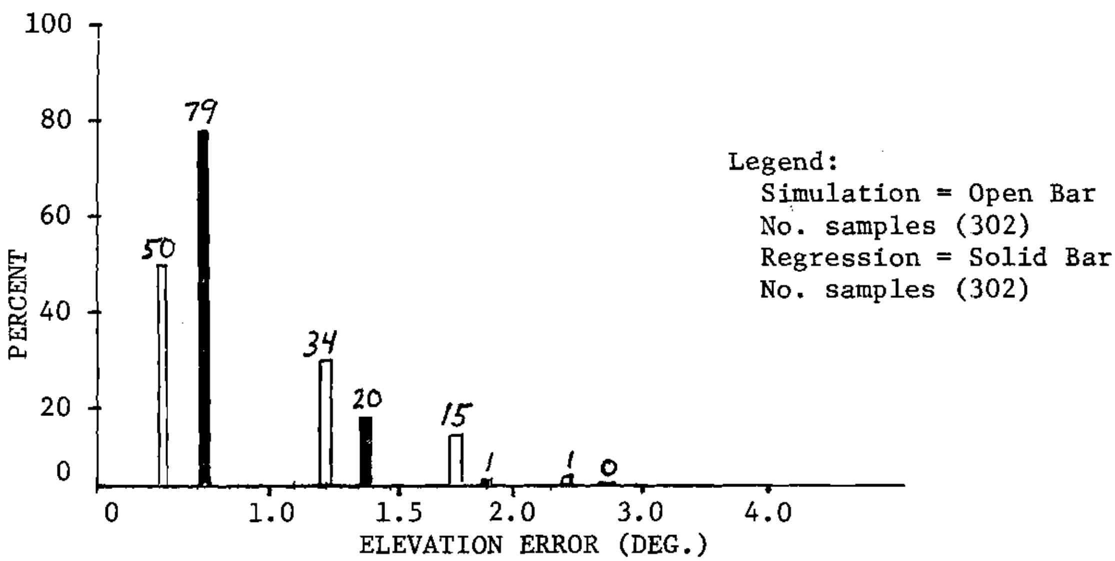

Figure 11. Absolute Elevation Error Distribution (Prediction Passes)

regression technique compatibility are presented to demonstrate the technical value of this technique. Likewise, cost factors dealing with data collection, model development and computer program execution are discussed to illustrate possible economic trade-offs.

A quantitative measure of performance must be available throughout the study to aid in making decisions concerning the calibration procedures. In this example, the 1.5 degree position error requirement supported the decision to reject the single pass calibration attempt in favor of the multiple pass approach that was finally used. Had the position errors not been reduced below this measure, further analysis would have been performed. Although this study had excellent results, there is of course no assurance that the calibration results will be accurate when extrapolated outside the range of velocity and acceleration values of this study. The robustness of this technique is limited by the data coverage.

A more subtle restriction on robustness relates to the actual system used to collect baseline data for comparison. The actual system may or may not be representative of the general population of such systems that may be encountered. Since it is not feasible to determine the true relation of the actual system to this population, further study would be needed if increased information suggested the actual system used in this study deviated significantly from the new information.

The accuracy of flight testing instrumentation represents a technical factor that directly impacts the utility of the regression technique. In this study the poor elevation parameter estimates are partially attributed to instrumentation errors. It is certain that, had these errors been much larger, this technique would have failed for the elevation model. In such a case, other alternatives to the regression technique would have been considered.

The minimal set of data passes necessary for proper calibration may vary considerably, especially if the model must be calibrated over large ranges of the independent variables. In this study, the minimal set of four passes was arrived at after realizing that one pass did not provide sufficient information. A more general procedure would be to start with a typical set of passes that included the spectrum of velocity/acceleration data from a set of candidates for calibration, and to keep adding passes until all other passes could be successfully estimated.

Technique compatibility, the final technical factor, refers to the interface requirements between the regression technique and the simulation program. In this study the simulation program produced time series data that facilitated fitting the regression equation to the position errors. However, if the simulation output had been other parameters such as projectile miss distances, or probability of kill, the regression technique could not have been used as designed. Additionally, if the data available from the actual and simulated systems did not contain information relevant to the original hypothesis, the regression technique would not be applicable.

Turning to the economic utility, the most costly factor was the model development which included study effort, programming effort, system analysis efforts, and computer utilization. Unless ample information is available about the actual system, much time will be required in studying input/output in developing the hypothesis needed to construct the regression model. In this study, much information was available from personnel who had written the simulation program, P2418. This fact together with the easy access to flight test data tended to minimize these costs. On the other hand, even though much model devleopment effort may be required, it may be less than the efforts required to gain more detailed information about the actual system. Computer time and programming effort were minimal for this study and should not be large for other applications unless the number of variables or the amount of calibration data are significantly increased.

The computer program execution costs and the additional costs of running the simulation program with the regression equation adjustments to the output were minimal for this study. The computer based cost associated with solving two linear equations with two variables is negligible. If the number of equations and/or variables significantly increased, the cost would not be significant when compared with the cost of the full simulation program.

For other applications of this technique, data collection could be the most costly economic factor. Typical cost for a flight test may exceed $2,000 per hour, so that collecting sufficient data making regression estimates may run into several multiples of this cost. Fortunately, the data for this study had been previously collected for a similar purpose and its collection cost was not a factor.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

The semi-empirical approach of applying regression analysis to calibrate existing engagement simulation programs using flight test data appears to be a viable methodology. In support of this, the application of regression analysis to an existing simulation, P2418, yielded high utility in increased accuracy, robustness over the conditions of this study, modest data requirements for calibration, compatibility with existing simulations, and modest model development time. The application markedly decreased simulated position errors. The percentage of azimuth errors exceeding 1.5 degrees was reduced from 76% to 0% and the percentage of elevation position errors exceeding 1.5 degrees was reduced from 16% to 1%.

This approach could be applied to other engagement simulations. Successful application would require the available additional data to meet adequacy standards as to relevance, accuracy, and coverage. The data used in this study were quite relevant, but their accuracy and coverage verged on being inadequate for calibration purposes. Since the present calibration was successful in spite of these difficulties, it is likely that many existing field tests are potentially useful for calibrating related simulations. The results of this study suggest that calibration usefulness would not be confined to tests that have yielded simulation-relevant data of extraordinary accuracy and coverage.

4.2 Recommendations

This study indicates that the accuracy of simulation program P2418 could be greatly enhanced by modifying the simulation output to take into account several characteristics shown by the flight test data to be exhibited by the actual system. These modifications include an azimuth velocity limit, azimuth position change proportional to azimuth velocity and acceleration, and elevation position change proportional to elevation velocity and acceleration.

It is therefore recommended that the following output modifications be incorporated into P2418:

1. P2418 does not exhibit an azimuth velocity limit as seen in the actual system. This is considered a significant modeling difference and the following statements should be added to P2418:

$$V_{az_i} = \min\left(\left|X_{az_i} - X_{az_{i-1}}\right|)10,\ 80\right)$$

$$\hat{X}_{az_i} = \hat{X}_{az_{i-1}} + ((V_{az_{i-1}})/10)S$$

where V_{az_i} = azimuth velocity commanded by P2418 or

80 degrees per second at time t_i

X_{az_i} = azimuth position of gun at time t_i

$\hat{X}_{az_i}$ = corrected azimuth position of gun

$S = (X_{az_{i-1}})/|X_{az_{i-1}}|$

Since the output of P2418 does not include velocity, the first statement simply computes the velocity using the difference in position.

2. The P2418 azimuth position leads the actual system due to velocity and acceleration drag not modeled. The output should be modified by adding the following statements:

$$\hat{X}_{az_i} = 0.121\ V_{az_i} + 0.003\ A_{az_i}$$

where V_{az_i} = azimuth velocity commanded by P2418 at time t_i

(i.e. $V_{az_i} = (X_{az_i} - X_{az_{i-1}})10$)

A_{az_i} = azimuth acceleration commanded by P2418 at time t_i

(i.e., $A_{az_i} = (V_{az_i} - V_{az_{i-1}})10$)

X_{az_i} = azimuth position of gun at time t_i (output of P2418)

$\hat{X}_{az_i}$ = corrected azimuth position of gun

3. Likewise the following statements should be added to alter the elevation position output:

$$\hat{X}_{e\ell_i} = 0.139\, V_{e\ell_i} + 0.025\, A_{e\ell_i}$$

where $V_{e\ell_i}$ = elevation velocity commanded by P2418 at time t_i (i.e., $V_{e\ell_i} = (X_{e\ell_i} - X_{e\ell_{i-1}})10$)

$A_{e\ell_i}$ = elevation acceleration commanded by P2418 at time t_i (i.e., $A_{e\ell_i} = (V_{e\ell_i} - V_{e\ell_{i-1}})10$)

$X_{e\ell_i}$ = elevation position of gun at time t_i (output of P2418)

$\hat{X}_{e\ell_i}$ = corrected elevation position of gun

The success of this methodology suggests that it should be applied to other engagement simulations exhibiting accuracy difficulties where test data (or other quantitative information) exists but has not been exploited during simulation development.

Field tests are usually designed for evaluating weapon systems with little or no consideration given to the potential usefulness of the resulting data in improving simulation of weapon systems used in the test. As this study has demonstrated, field test data that was collected for unrelated purposes can provide calibration data for simulations of the associated weapon system used in the test. It is recommended that this potential be considered as a standard procedure during test design.

APPENDICES

APPENDIX A

This appendix briefly describes the data collection activities that produced data for this study. The activities discussed relate to the flight test profiles, data collection, and data reduction.

Flight testing consisted of several data collection passes made by a high performance aircraft over the ground-based antiaircraft artillery system that was being tested. Airspeed was about 400 miles per hour and altitude ranged between 300 and 1,500 feet. The offset varied from directly over the ground site to about 2,000 feet offset to the South. All passes were made straight-and-level with either eastbound or westbound headings. Airspeed was approximately 370 knots true airspeed. Data collection started for each pass when the aircraft was inbound at approximately 10,000 feet slant range and terminated shortly after the aircraft passed over the site.

Figure A-1 illustrates the data collection points from the actual system. These data were collected at 10 samples per second. Tape 1 recorded aircraft azimuth, elevation, and range relative to the ground site. Tape 2 recorded gun elevation and azimuth using the same reference. During the simulation run, tape 1 data were used as input and a tape 3, similar to tape 2, was recorded that contained simulated gun position data. Subsequently, tapes 2 and 3 were input to a reduction program to compute the time correlated differences between the simulated and actual gun positions. Tape 3 data were also differenced twice to produce velocity and acceleration time series data. The time correlated differences between tapes 2 and 3 were used as the dependent variable, e, in the regression analysis. The first and second differences of tape 3 were used as the velocity and acceleration independent variables, X_1 and X_2.

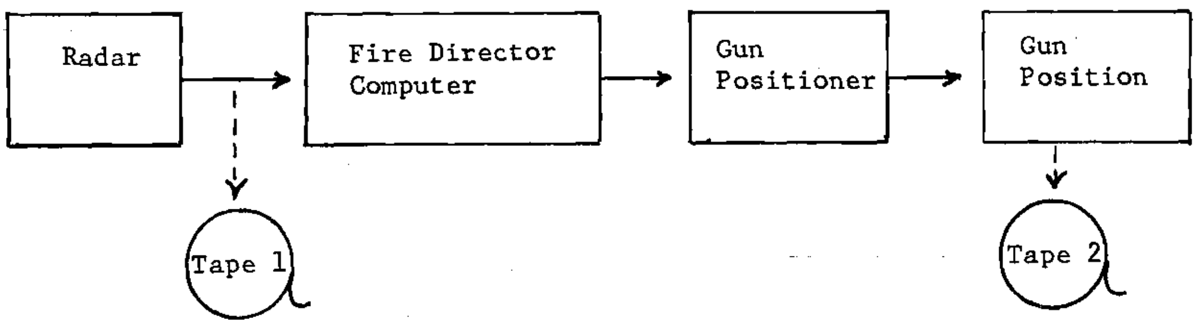

Figure A-1. Data Collection From Actual System

APPENDIX B

This appendix contains the data used in computing the regression estimates. The following data are included for passes 4 through 7:

1. Azimuth Velocity
2. Azimuth Acceleration
3. Elevation Velocity
4. Elevation Acceleration
5. Simulated gun position error for both azimuth and elevation.

Table B-1. Pass 4 Data

Azimuth

Velocity (deg/sec)	Acceleration (deg/sec/sec)	Difference (y) (deg)
-10.548	-13.337	-.978
-11.819	-12.709	-.973
-12.919	-10.992	-.947
-14.019	-11.002	-.986
-15.510	-14.910	-1.131
-17.794	-22.847	-1.373
-21.053	-32.587	-1.676
-25.357	-43.037	-2.014
-30.838	-54.812	-2.505
-37.612	-67.744	-3.103
-45.613	-80.007	-4.016
-54.822	-92.088	-5.148
-65.297	-104.746	-6.668
-76.907	-116.108	-8.690
-89.247	-123.400	-11.198

Elevation

Velocity (deg/sec)	Acceleration (deg/sec/sec)	Difference (y) (deg)
11.933	.019	1.354
12.488	5.552	1.505
13.465	9.767	1.6v5
13.648	1.827	1.689
13.891	2.434	1.804
14.246	3.543	1.756
14.559	3.135	1.828
14.810	2.506	1.880
14.535	-2.745	1.774
13.820	-7.150	1.838
12.716	-11.038	1.703
11.346	-13.698	1.497
9.680	-16.660	1.191
7.664	-20.168	.749
5.373	-22.912	.341

Table B-2. Pass 5 Data

Velocity (deg/sec)	Azimuth Acceleration (deg/sec/sec)	Difference (y) (deg)
-10.173	-4.670	-1.145
-10.625	-4.520	-1.153
-11.021	-3.961	-1.200
-11.317	-2.957	-1.233
-11.568	-2.504	-1.248
-11.843	-2.759	-1.289
-12.149	-3.054	-1.318
-12.406	-2.571	-1.328
-12.590	-1.836	-1.356
-12.917	-3.277	-1.374
-13.487	-5.697	-1.448
-14.066	-5.794	-1.448
-14.592	-5.252	-1.413
-15.099	-5.076	-1.429
-15.466	-3.673	-1.438
-15.648	-1.818	-1.464
-15.762	-1.134	-1.502
-15.991	-2.292	-1.607
-16.312	-3.215	-1.745
-16.590	-2.773	-1.909
-16.789	-1.998	-1.874
-17.046	-2.561	-1.821
-17.477	-4.318	-1.899
-18.044	-5.663	-1.989
-18.626	-5.824	-1.875
-19.040	-4.137	-1.757
-19.232	-1.921	-1.835
-19.188	.436	-1.952
-18.921	2.668	-2.042
-18.500	4.216	-2.046
-17.954	5.456	-2.084
-17.390	5.639	-2.153
-16.985	4.049	-2.269
-16.877	1.087	-2.419
-17.004	-1.273	-2.046
-17.164	-1.606	-2.364
-17.265	-1.005	-2.245
-17.260	.052	-2.169

Table B-2 cont'd.

Velocity (deg/sec)	Azimuth Acceleration (deg/sec/sec)	Difference (y) (deg)
-17.094	1.654	-2.077
-16.901	1.936	-2.009
-16.434	4.666	-2.114
-15.859	5.746	-2.250
-15.447	4.125	-2.389
-15.066	3.807	-2.445
-14.111	9.548	-2.406
-13.239	8.726	-2.280
-13.158	.812	-2.189
-12.889	2.690	-2.116
-12.622	2.663	-2.104
-12.539	.838	-2.127
-12.723	-1.848	-2.125
-12.663	.604	-2.029
-12.369	2.944	-1.904
-11.920	4.491	-2.041
-11.286	6.392	-2.070
-10.493	7.871	-2.021

Velocity (deg/sec)	Elevation Acceleration (deg/sec/sec)	Difference (y) (deg)
3.227	.389	-.761
3.341	1.139	-.778
3.476	1.354	-.716
3.459	-.173	-.700
3.093	-3.654	-.720
2.712	-3.809	-.735
2.632	-.807	-.801
2.675	.438	-.819
2.698	.229	-.813
2.926	2.276	-.828
3.125	1.996	-.779
2.955	-1.706	-.747
2.871	-.840	-.746
2.812	-.593	-.728
2.444	-3.675	-.726
2.234	-2.104	-.766
2.097	-1.369	-.754
1.945	-1.519	-.801

Table B-2 cont'd.

Velocity (deg/sec)	Elevation Acceleration (deg/sec/sec)	Difference (y) (deg)
2.009	.638	-.776
1.914	-.943	-.783
1.676	-2.380	-.813
1.559	-2.871	-.784
.949	-4.405	-.865
.399	-5.499	-.847
-.169	-5.675	-.951
-.681	-5.119	-1.063
-1.128	-4.469	-1.154
-1.313	-1.859	-1.198
-1.274	.390	-1.281
-1.307	-.322	-1.280
-1.501	-1.939	-1.210
-1.645	-1.445	-1.243
-1.779	-1.340	-1.267
-1.994	-2.153	-1.205
-2.388	-3.716	-1.242
-r.882	-5.158	-1.288
-3.227	-3.457	-1.215
-3.347	-1.196	-1.242
-3.405	-.579	-1.253
-3.427	-.217	-1.222
-3.529	-1.026	-1.180
-3.813	-2.838	-1.232
-4.127	-3.141	-1.227
-4.312	-1.851	-1.175
-4.241	.711	-1.203
-4.390	-1.485	-1.203
-4.867	-4.771	-1.206
-5.114	-2.472	-1.278
-5.433	-3.191	-1.382
-5.779	-3.457	-1.345
-5.884	-1.054	-1.406
-5.887	-.026	-1.423
-5.837	.494	-1.479
-5.842	-.047	-1.404
-5.774	.681	-1.454
-5.607	1.664	-1.532

Table B-3. Pass 6 Data

Velocity (deg/sec)	Azimuth Acceleration (deg/sec/sec)	Difference (y) (deg)
-10.499	-5.972	-1.683
-11.043	-5.440	-1.776
-11.803	-7.604	-1.990
-13.128	-13.248	-2.204
-15.287	-21.588	-2.546
-18.622	-33.351	-2.870
-23.508	-48.862	-3.288
-30.041	-65.325	-3.831
-38.253	-82.123	-4.624
-48.394	-101.411	-5.772
-60.890	-124.960	-7.422
-76.550	-156.604	-9.936
-95.322	-187.713	-13.491
-115.995	-206.735	-18.104

Velocity (deg/sec)	Elevation Acceleration (deg/sec/sec)	Difference (y) (deg)
12.967	1.235	1.644
13.228	2.610	1.670
13.881	6.531	1.740
15.032	11.505	1.837
16.006	9.749	1.965
16.160	4.539	2.315
16.729	2.682	2.252
16.521	-2.080	2.366
15.777	-7.436	2.361
14.639	-11.384	2.001
13.160	-14.788	1.867
11.137	-20.230	1.443
7.206	-39.310	.801
2.442	-47.635	.035

Table B-4. Pass 7 Data

Velocity (deg/sec)	Azimuth Acceleration (deg/sec/sec)	Difference (y) (deg)
-10.547	-10.029	-.966
-11.328	-7.807	-.957
-11.797	-4.692	-.950
-12.027	-2.298	-1.010
-12.010	.167	-1.068
-11.868	1.422	-1.112
-11.837	.303	-1.242
-12.049	-2.112	-1.436
-12.464	-4.152	-1.496
-12.982	-5.181	-1.387
-13.564	-5.816	-1.294
-14.190	-6.265	-1.306
-14.785	-5.950	-1.291
-15.262	-4.766	-1.279
-15.609	-3.472	-1.302
-15.782	-1.728	-1.342
-15.921	-1.391	-1.440
-16.356	-4.352	-1.581
-17.022	-6.623	-1.745
-17.536	-5.133	-1.785
-17.694	-1.581	-1.709
-17.715	-.212	-1.678
-17.917	-2.016	-1.844
-18.342	-4.259	-1.964
-18.805	-4.623	-1.911
-19.147	-3.425	-1.804
-19.320	-1.726	-1.935
-19.352	-.321	-2.068
-19.263	.887	-2.061
-19.020	2.428	-2.073
-18.708	3.129	-2.186
-18.444	2.637	-2.317
-18.230	2.139	-2.426
-17.997	2.327	-2.380
-17.659	3.386	-2.300
-17.288	3.712	-2.359

Table B-4 cont'd. (Azimuth)

-17.004	2.831	-2.433
-16.877	1.271	-2.363
-16.540	3.372	-2.128
-16.014	5.263	-2.191
-15.727	2.865	-2.270
-15.646	.808	-2.208
-15.341	3.050	-2.116
-14.851	4.900	-2.151
-14.516	3.358	-2.284
-14.344	1.714	-2.269
-14.119	2.252	-2.186
-13.478	6.411	-2.040
-13.121	3.564	-2.054
-12.801	3.201	-2.127
-12.289	5.126	-2.082
-12.001	2.881	-1.964
-11.843	1.578	-1.830
-11.747	.956	-1.862
-11.590	1.570	-1.878
-11.301	2.894	-1.910
-10.860	4.409	-1.855
-10.333	5.268	-1.832

Table B-4. Pass 7 Data

Velocity (deg/sec)	Elevation Acceleration (deg/sec/sec)	Difference (y) (deg)
4.225	.159	-.7049
3.670	-5.549	-.6455
3.211	-4.590	-.7199
3.105	-1.066	-.7609
2.965	-1.392	-.7720
2.707	-2.587	-.8529
2.788	.811	-.8598
3.123	3.352	-.8770
3.219	.961	-.8627
3.102	-1.169	-.8381
2.807	-2.958	-.8431
2.351	-4.552	-.8936
2.285	-.663	-.9288
2.426	1.414	-.9059
2.443	.163	-.9033
2.682	2.395	-.8546
2.764	.814	-.7762
2.255	-5.091	-.7705
1.460	-7.945	-.7783
1.056	-4.038	-.8704
1.395	3.387	-.8627
1.521	1.257	-.8205
1.002	-5.184	-.8302
.454	-5.479	-.8946
.131	-3.235	-.9254
-.171	-3.022	-.9865
-.752	-5.309	-1.0617
-1.287	-5.352	-1.1685
-1.527	-2.392	-1.2553
-1.711	-1.839	-1.3165
-1.688	.229	-1.3314
-1.837	-1.494	-1.2515
-2.331	-4.940	-1.2429
-2.559	-2.276	-1.3449
-2.534	.248	-1.2687
-2.617	-.834	-1.2228
-2.939	-3.219	-1.2311
-3.157	-2.179	-1.2612
-3.288	-1.394	-1.2384
-3.516	-2.286	-1.2164
-3.565	-.485	-1.2213
-3.494	.703	-1.1753
-3.607	-1.127	-1.1625
-4.061	-4.535	-1.1071
-4.813	-7.521	-1.2807
-5.423	-6.107	-1.3836
-5.452	-.287	-1.2916
-5.517	-.652	-1.3160
-5.920	-4.029	-1.4466
-5.967	-.463	-1.4720
-6.034	-.670	-1.4161
-6.394	-3.603	-1.4842
-6.422	-.285	-1.5112
-6.190	2.327	-1.5589
-6.070	1.198	-1.5946
-6.000	.697	-1.5794
-5.756	2.441	-1.6057
-5.548	2.080	-1.5672

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